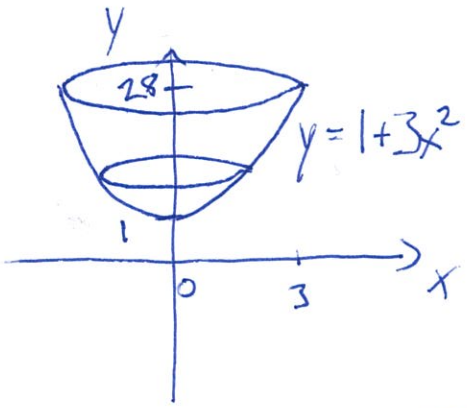
MATH 2260 Calculus II
Eric Perkerson

Name:
Date: June 25, 2015

Quiz No. 2

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 (25 Points) Find the area of the surface generated by revolving the curve $y = 1 + 3x^2$ from $x = 0$ to $x = 3$ about the y-axis. (Hint: Yes, you have to invert this function.)



$y = 1+3x^2 \Rightarrow \frac{y-1}{3} = x^2 \Rightarrow x = \sqrt{\frac{y-1}{3}}$

$y(0) = 1$, $y(3) = 1+3(3^2) = 28$

$$S = \int_1^{28} 2\pi \sqrt{\frac{y-1}{3}} \sqrt{1 + \left(\frac{d}{dy}\left(\sqrt{\frac{y-1}{3}}\right)\right)^2}\, dy$$

$$= 2\pi \int_1^{28} \sqrt{\frac{y-1}{3}} \sqrt{1 + \left(\frac{1}{2}\left(\frac{y-1}{3}\right)^{-1/2}\left(\frac{1}{3}\right)\right)^2}\, dy$$

$$= 2\pi \int_1^{28} \sqrt{\frac{y-1}{3}} \sqrt{1 + \frac{1}{36}\left(\frac{y-1}{3}\right)^{-1}}\, dy$$

$$= 2\pi \int_1^{28} \sqrt{\frac{y-1}{3} + \frac{1}{36}}\, dy$$

$$= 2\pi \int_1^{28} \frac{1}{\sqrt{3}} \sqrt{y-1+\frac{1}{13}}\, dy$$ 20 pt to here, $u = y-1+\frac{1}{13}$, $du = dy$

$$= \frac{2\pi}{\sqrt{3}} \int_1^{28} \sqrt{u}\, du = \frac{2\pi}{\sqrt{3}} \left[\frac{u^{3/2}}{3/2}\right]_1^{28} = \frac{2\pi}{\sqrt{3}}\left(\frac{2}{3}\right)\left[\left(y-1+\frac{1}{13}\right)^{3/2}\right]_1^{28}$$

$$= \frac{4\pi}{3\sqrt{3}}\left(y-1+\frac{1}{13}\right)^{3/2}\Big|_1^{28}$$

To here
5 pt
-1 for each incorrect bound
5 pt if volume instead of s.a
5 pt for trying

Problem 2 *(25 Points) A force of 4 pounds is required to hold a spring stretched* 0.1 *feet beyond its natural length. How much work is done in stretching the spring from its natural length to 1.1 feet beyond its natural length?*

$$F = kx \Rightarrow 4 = k\left(\frac{1}{10}\right) \Rightarrow k = 40$$

$$W = \int_0^{1.1} F(x)\,dx = \int_0^{1.1} 40x\,dx = 40\left[\frac{x^2}{2}\right]_0^{1.1}$$

$$= 20(1.1)^2$$

-3 for 1 instead of 1.1

-2 arithmetic errors

Problem 3 *(25 Points) Find an equation of the curve that satisfies*

$$\frac{dy}{dx} = 24yx^{11}$$

whose y-intercept is 6.

$$\frac{dy}{dx} = 24yx^{11}$$

$$\int \frac{1}{y}\,dy = \int 24x^{11}\,dx$$

$$\ln(|y|) = 24\left[\frac{x^{12}}{12}\right] + C$$

$$\ln(|y|) = 2x^{12} + C$$

$$|y| = e^{2x^{12}+C}$$

$$|y| = e^{C}e^{2x^{12}}$$

And we know $y(0) = 6$

$$\Rightarrow y = e^{C}e^{2(0)} = 6 \Rightarrow e^{C} = 6 \Rightarrow C = \ln(6)$$

$$\Rightarrow y(x) = 6e^{2x^{12}}$$

-5 for wrong intercept, -10 no ln

Problem 10 *(25 Points) Evaluate the integral*

$$\int (\log(x))^2 dx$$

$$\int \log^2(x)\,dx = \int \underbrace{(1)}_{f'} \underbrace{\log^2(x)}_{g}\,dx = x\log^2(x) - \int x\left(2\log(x)\tfrac{1}{x}\right)dx$$

$$= x\log^2(x) - 2\int \log(x)\,dx$$

$$= x\log^2(x) - 2\int \underbrace{(1)}_{u'} \underbrace{\log(x)}_{v}\,dx$$

$$= x\log^2(x) - 2\left(x\log(x) - \int x\tfrac{1}{x}\,dx\right)$$

$$= x\log^2(x) - 2x\log(x) + 2x + C$$